

# derivation of Schrodinger Equation

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## Abstract

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Eq. 1 is an initial equation.

$$k = \frac{2\pi}{\lambda} \quad (1)$$

Eq. 2 is an initial equation.

$$\omega = 2\pi f \quad (2)$$

Eq. 3 is an initial equation.

$$\hbar = h/(2\pi) \quad (3)$$

Eq. 4 is an initial equation.

$$p = h/\lambda \quad (4)$$

Eq. 5 is an initial equation.

$$E = hf \quad (5)$$

Divide both sides of Eq. 2 by  $2\pi$ ; yields Eq. 6.

$$\frac{\omega}{2\pi} = f \quad (6)$$

Substitute RHS of Eq. 6 into Eq. 5; yields Eq. 7.

$$E = \frac{h\omega}{2\pi} \quad (7)$$

Substitute RHS of Eq. 3 into Eq. 7; yields Eq. 8.

$$E = \omega\hbar \quad (8)$$

Divide both sides of Eq. 8 by  $\hbar$ ; yields Eq. 9.

$$\frac{E}{\hbar} = \omega \quad (9)$$

Divide both sides of Eq. 1 by  $2\pi$ ; yields Eq. 10.

$$\frac{k}{2\pi} = \lambda \quad (10)$$

Substitute RHS of Eq. 4 into Eq. 10; yields Eq. 11.

$$p = \frac{hk}{2\pi} \quad (11)$$

Substitute RHS of Eq. 11 into Eq. 3; yields Eq. 12.

$$p = \hbar k \quad (12)$$

Divide both sides of Eq. 12 by  $\hbar$ ; yields Eq. 13.

$$\frac{p}{\hbar} = k \quad (13)$$

Replace scalar variables in Eq. 13 with equivalent vector variables; yields Eq. 14.

$$\frac{\vec{p}}{\hbar} = \vec{k} \quad (14)$$

Eq. 15 is an initial equation.

$$\psi(\vec{r}, t) = \psi_0 \exp \left( i \left( \vec{k} \cdot \vec{r} - \omega t \right) \right) \quad (15)$$

Substitute RHS of Eq. 15 into Eq. 14; yields Eq. 16.

$$\psi(\vec{r}, t) = \psi_0 \exp \left( i \left( \frac{\vec{p} \cdot \vec{r}}{\hbar} - \omega t \right) \right) \quad (16)$$

Substitute RHS of Eq. 16 into Eq. 9; yields Eq. 17.

$$\psi(\vec{r}, t) = \psi_0 \exp \left( i \left( \frac{\vec{p} \cdot \vec{r}}{\hbar} - \frac{Et}{\hbar} \right) \right) \quad (17)$$

Simplify Eq. 17; yields Eq. 18.

$$\psi(\vec{r}, t) = \psi_0 \exp \left( \frac{i}{\hbar} (\vec{p} \cdot \vec{r} - Et) \right) \quad (18)$$

Eq. 19 is an initial equation.

$$p = mv \quad (19)$$

Eq. 20 is an initial equation.

$$E = \frac{1}{2}mv^2 \quad (20)$$

Raise both sides of Eq. 19 to 2; yields Eq. 21.

$$p^2 = m^2v^2 \quad (21)$$

Multiply RHS of Eq. 20 by 1, which in this case is  $m/m$ ; yields Eq. 22

$$E = \frac{1}{2m}m^2v^2 \quad (22)$$

Substitute RHS of Eq. 21 into Eq. 22; yields Eq. 23.

$$E = \frac{p^2}{2m} \quad (23)$$

Partially differentiate Eq. 18 with respect to  $t$ ; yields Eq. 24.

$$\frac{\partial}{\partial t} \psi(\vec{r}, t) = \psi_0 \frac{\partial}{\partial t} \exp \left( i \left( \frac{\vec{p} \cdot \vec{r}}{\hbar} - \frac{Et}{\hbar} \right) \right) \quad (24)$$

Substitute RHS of Eq. 24 into Eq. 18; yields Eq. 25.

$$\frac{\partial}{\partial t} \psi(\vec{r}, t) = \frac{-i}{\hbar} E \psi(\vec{r}, t) \quad (25)$$

Substitute RHS of Eq. 23 into Eq. 25; yields Eq. 26.

$$\frac{\partial}{\partial t} \psi(\vec{r}, t) = \frac{-i}{\hbar} \frac{p^2}{2m} \psi(\vec{r}, t) \quad (26)$$

Multiply both sides of Eq. 26 by  $i\hbar$ ; yields Eq. 27.

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = \frac{p^2}{2m} \psi(\vec{r}, t) \quad (27)$$

Apply gradient to both sides of Eq. 18; yields Eq. 28.

$$\psi(\vec{r}, t) = \psi_0 \exp \left( \frac{i}{\hbar} (\vec{p} \cdot \vec{r} - Et) \right) \quad (28)$$

Simplify Eq. 28; yields Eq. 29.

$$\vec{\nabla} \psi(\vec{r}, t) = \frac{i}{\hbar} \vec{p} \psi_0 \exp \left( \frac{i}{\hbar} (\vec{p} \cdot \vec{r} - Et) \right) \quad (29)$$

Substitute RHS of Eq. 29 into Eq. 18; yields Eq. 30.

$$\vec{\nabla} \psi(\vec{r}, t) = \frac{i}{\hbar} \vec{p} \psi(\vec{r}, t) \quad (30)$$

Apply divergence to both sides of Eq. 30; yields Eq. 31.

$$\vec{\nabla} \cdot \left( \vec{\nabla} \psi(\vec{r}, t) \right) = \frac{i}{\hbar} \vec{\nabla} \cdot (\vec{p} \psi(\vec{r}, t)) \quad (31)$$

Simplify Eq. 31; yields Eq. 32.

$$\nabla^2 \psi(\vec{r}, t) = \frac{i}{\hbar} \vec{p} \cdot \left( \vec{\nabla} \psi(\vec{r}, t) \right) \quad (32)$$

Substitute RHS of Eq. 30 into Eq. 32; yields Eq. 33.

$$\nabla^2 \psi(\vec{r}, t) = \frac{i}{\hbar} \vec{p} \cdot \left( \frac{i}{\hbar} \vec{p} \psi(\vec{r}, t) \right) \quad (33)$$

Simplify Eq. 33; yields Eq. 34.

$$\nabla^2 \psi(\vec{r}, t) = \frac{-p^2}{\hbar} \psi(\vec{r}, t) \quad (34)$$

Multiply both sides of Eq. 34 by  $\frac{-\hbar^2}{2m}$ ; yields Eq. 35.

$$\frac{-\hbar^2}{2m} \nabla^2 \psi(\vec{r}, t) = \frac{p^2}{2m} \psi(\vec{r}, t) \quad (35)$$

LHS of Eq. 27 is equal to LHS of Eq. 35; yields Eq. 36.

$$\frac{-\hbar^2}{2m} \nabla^2 \psi(\vec{r}, t) = i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) \quad (36)$$

Eq. 37 is an initial equation.

$$\frac{-\hbar^2}{2m} \nabla^2 = \mathcal{H} \quad (37)$$

Substitute LHS of Eq. 37 into Eq. 36; yields Eq. 38.

$$\mathcal{H} \psi(\vec{r}, t) = i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) \quad (38)$$

Eq. 38 is one of the final equations.

## References