equation of motion for a spring

none

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Abstract

Generated by the Physics Derivation Graph.

Eq. 1 is an initial equation.

$$F = ma \tag{1}$$

Eq. 2 is an initial equation.

$$F_{\text{spring}} = -kx \tag{2}$$

Eq. 3 is an initial equation.

$$a = \frac{d^2x}{dt^2} \tag{3}$$

LHS of Eq. 1 is equal to LHS of Eq. 2; yields Eq. 4.

$$ma = -kx \tag{4}$$

Divide both sides of Eq. 4 by m; yields Eq. 5.

$$a = -\frac{k}{m}x \tag{5}$$

LHS of Eq. 5 is equal to LHS of Eq. 3; yields Eq. 6.

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \tag{6}$$

Judicious choice as a guessed solution to Eq. 6 is Eq. 7, what, when differentiated twice, yields a negative of itself? cosine

$$x(t) = A\cos(\omega t) \tag{7}$$

Differentiate Eq. 7 with respect to t; yields Eq. 8.

$$\frac{dx}{dt} = -A\omega\sin(\omega t) \tag{8}$$

Differentiate Eq. 8 with respect to t; yields Eq. 9.

$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t) \tag{9}$$

LHS of Eq. 9 is equal to LHS of Eq. 6; yields Eq. 10.

$$-\frac{k}{m}x = -A\omega^2\cos(\omega t) \tag{10}$$

Substitute LHS of Eq. 7 into Eq. 10; yields Eq. 11.

$$-\frac{k}{m}A\cos(\omega t) = -A\omega^2\cos(\omega t) \tag{11}$$

Multiply both sides of Eq. 11 by $\frac{-1}{A\cos(\omega t)}$; yields Eq. 12.

$$\frac{k}{m} = \omega^2 \tag{12}$$

Take the square root of both sides of Eq. 12; yields Eq. 13 and Eq. 14.

$$\sqrt{\frac{k}{m}} = \omega \tag{13}$$

$$-\sqrt{\frac{k}{m}} = \omega \tag{14}$$

Substitute RHS of Eq. 13 into Eq. 7; yields Eq. 15.

$$x(t) = A\cos\left(\frac{k}{m}t\right) \tag{15}$$

Eq. 15 is one of the final equations.

References